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Experiments on the flow past an inclined disk

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The wake of a disk at an angle to a stream contains marked periodic motions which arise from the regular shedding of vortices from the trailing edge. The vortices are in the form of a chain of irregular rings, each one linked to the succeeding one, and they move downstream at about 0·6 of free-stream velocity. The prominence of the vortex shedding increases as the angle of incidence (measured from the normal) increases up to at least 50°. The shedding frequency increases with the angle of incidence, but by a suitable choice of reference velocity and length scale, may be described by a wake Strouhal number which has the constant value 0·21 for all angles of incidence above zero, up to at least 40°.

Axially-symmetric bodies at zero incidence shed vortices in a similar manner, except that the orientation of the plane of vortex shedding is not fixed and varies from time to time.

1. Introduction

In the wakes of most bluff bodies it is possible to detect periodic changes of velocity. These correspond to the regular shedding of vortices which arise from instability of the separated shear layers. In two-dimensional configurations these can dominate the wake, but in simple axisymmetric arrangements the periodicity is usually rather weak (Calvert 1967). If, however, a disk is inclined so that its normal makes an angle between 10° and 50° with the stream direction very strong vortex shedding occurs. This effect does not appear to have been investigated, or indeed commented upon, to any extent, although Eden (1912) has published photographs of the rather similar vortex shedding from an inclined square plate. Hoerner (1965) quotes results which show that the normal force coefficient for an inclined disk remains almost constant at 1·17 over the incidence range 0° to 45° from the normal, and is equal to that of an inclined square plate; apart from this there seems to be little information available on the flow past such bodies.

2. Experimental technique

Smoke flow visualization experiments were carried out in an open-return wind tunnel with a working section 12 by 6 in. The model was a 1 in. diameter disk 0·05 in. thick. Other experiments were made in an open-return wind tunnel with a working section 20 by 28 in. The model was then a 2 in. diameter $\frac{1}{16}$ in.

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thick disk, with its downstream edge bevelled at 45°. It was supported from downstream on a $\frac{1}{8}$ in. diameter sting which was held on the centre line of the wind tunnel by an arrangement of 0·015 in. piano wires. The support system is sketched in figure 1.

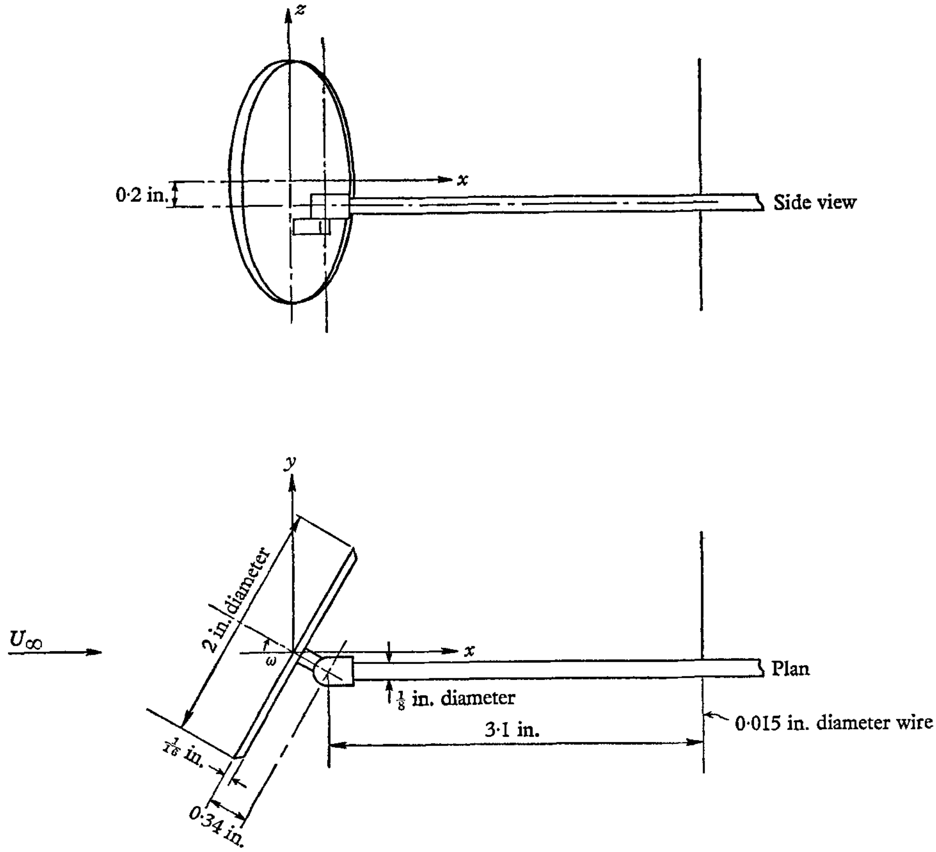

FIGURE 1. Sketch of model.

Velocity and turbulence measurements were made with a DISA 55A01 constant-temperature hot-wire anemometer. The assumptions made as to the response of this instrument are the same as in an earlier paper (Calvert 1967). The quantities obtained are U/U_∞, the ratio of the component of local mean velocity in the free-stream direction to the velocity at a reference point, and

$$\theta = (\overline{u^2})^{\frac{1}{2}}/U_\infty.$$

Static pressures were measured with a static tube aligned in the free-stream direction and thus will, in general, be subject to errors arising from local flow deviation. There will also be errors due to turbulence. No attempt was made to correct for either of these effects.

Base pressures were measured with a Pitot tube held close to the base; this technique is further discussed in §3.

Shedding frequencies were measured on frequency spectra, obtained from a frequency analyser and X–Y plotter as described previously (Calvert 1967).

Although efforts were made to minimize blockage and interference, it was not possible to eliminate them entirely. The correction for wind tunnel blockage will be about 0·8 % on velocity for the disk normal to the stream. For the disk inclined to the stream it is uncertain how the blockage should be calculated, but it will be of the same order. This value is reasonably small, so that blockage might be expected to affect slightly the magnitude of various measured quantities but not the general form of the wake. Measurements were primarily being made of the wake itself, rather than of the relationship between the wake and the model, so that blockage would not be expected to affect any conclusions reached. It is not possible to calculate the magnitude of the interference due to the traverse gear and support system, but it is thought to be small and its effect should be similar to that of blockage.

Efforts were made to keep the traverse gear and support configurations the same for all experiments, but the blockage will vary as the drag of the model varies with the angle of incidence. All results quoted are uncorrected for blockage.

The origin of co-ordinates is at the centre of the upstream face of the disk. The x axis is in the free-stream direction increasing downstream, the y axis is positive to the left looking downstream and the z axis is upwards. The disk was inclined about the z axis. The angle of incidence was taken as zero with the disk normal to the stream and positive when the trailing edge was on the positive y side.

Except for the base pressure measurements and the hot wire traverses mentioned at the end of § 3, all measurements were made in the (x, y)-plane.

Visualization experiments using paraffin smoke were carried out with the 1 in. diameter disk at Reynolds numbers (based on diameter) between 500 and 1000. Measurements of shedding frequency, velocity, turbulence and pressure were made in the wake of the 2 in. diameter disk at Reynolds numbers between $3{\cdot}5 \times 10^4$ and $5{\cdot}0 \times 10^4$.

3. Results

Smoke flow visualization experiments

At angles of incidence of more than about 20° strong vortex shedding was visible. As the angle was increased, the vortices became clearer and the shedding frequency rose. Photographs taken in the (x, y)- and (x, z)-planes (not simultaneously) of the flow past a disk at about 50° incidence are shown in figure 2, plate 1.

The flow pattern may be seen to consist of vortex loops shed from the trailing edge of the disk. These loops are not planar or continuous; each is linked to the next one behind by two filaments from the leading edge side.

The pattern is similar to that behind a square plate, photographed by Eden (1912) and to that at a lower Reynolds number behind a disk at zero incidence, photographed by Stanton & Marshall (1932). It agrees with the predictions of Rosenhead (1932) except that in this case the orientation of the vortex loops is defined by the incidence of the disk. It has little in common with the two-dimensional vortex street, and might be better described as a vortex chain.

It seems likely that the flow pattern at zero incidence is similar, except that the orientation of the vortices becomes indeterminate. Vortices may be shed from one point on the circumference for a few cycles, and then move to another; the change is probably produced by slight variations in the direction of the free stream. There would be no definite phase relationship between the various bursts of shedding, so that the characteristic frequency would be rather difficult to detect, as previously observed (Calvert 1967). This flow pattern is very similar to that deduced, on semi-theoretical grounds, by Rosenhead (1932).

When the disk is inclined the front stagnation point is displaced towards the leading edge. The point on the rear surface at which the reversed flow divides is displaced towards the trailing edge and at angles of incidence above about 15° it leaves the surface entirely. The flow over the rear surface is then everywhere towards the leading edge. This flow, originating from the leading edge separation, is comparatively steady and persists throughout the shedding cycle. The flow from the trailing edge separation forms a much tighter vortex and it is this vortex which is periodically shed. The flow pattern is sketched in figure 3. (Tests with a down tuft at the Reynolds number of the experiments described below indicated that the flow pattern was similar to that at low Reynolds number.)

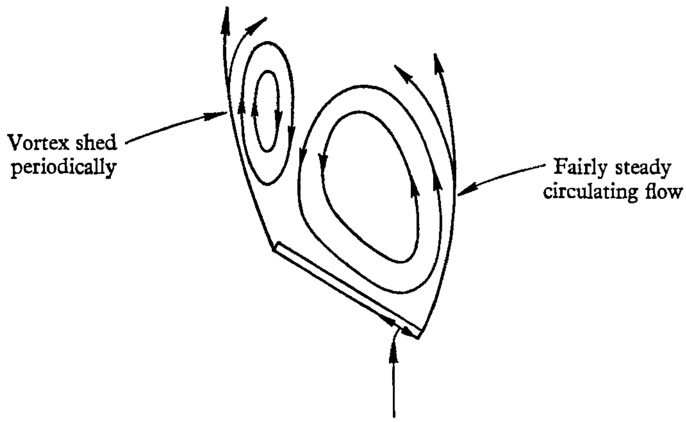

FIGURE 3. Sketch of flow immediately behind disk.

It was observed at various angles of incidence that the velocity with which the vortices moved downstream was about 0·6 of the free-stream velocity.

Shedding frequency

As the incidence is increased, the vortex shedding becomes more regular. Figure 4 shows frequency spectra at three different incidences, obtained by drawing a mean line through the band produced by the $X-Y$ plotter. (The thickness of this band is usually about 0·5 amplitude units.) The amplitude scales are not the same on the three curves, they were adjusted to give about the same maxima. Direct comparison of magnitudes is not very useful as it is impossible to be certain that the measurements were at corresponding points in the wake, but the height of the peak in relation to the background turbulence gives a measure of the relative importance of the vortex shedding.

Some spectra at angles of incidence of 30° and above have a second peak at twice the fundamental frequency. This probably arises from the inability of the hot-wire anemometer to distinguish between velocities in opposite directions. When the shedding is strong there will be a definite reversed flow over part of the cycle. Thus in certain regions of the wake the anemometer will record two velocity maxima per cycle. If the shedding is weak, this effect will be masked by background turbulence.

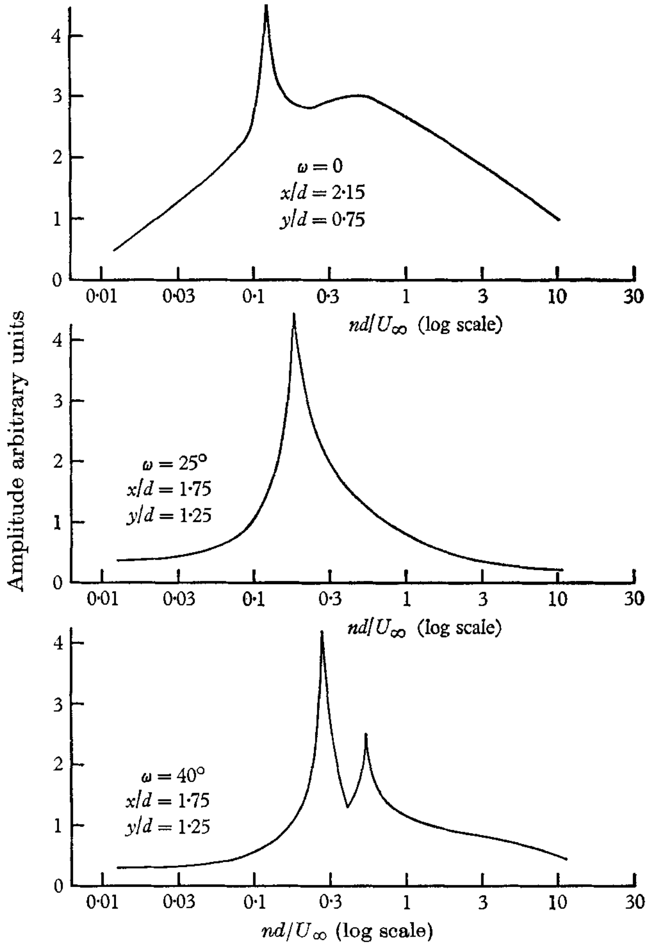

FIGURE 4. Frequency spectra in wakes of inclined disk.

At angles of incidence above about 25° it is possible to obtain the shedding frequency directly from the analyser meter without plotting a spectrum, and the shedding becomes obvious on the trace of an oscilloscope.

The peak is visible on the spectrum throughout the wake, but the clearest signal seems to be obtained just outside the wake on the trailing edge side, about 1·5 to 2 diameters downstream of the disk. The frequency is the same throughout the wake. The Strouhal number (nd/U_∞ at the peak) rises steadily with incidence; its variation is given in table 1 and plotted on figure 5.

If a square plate is inclined about an axis parallel to two of its sides the variation of Strouhal number with incidence is very similar to that of the disk if based on an equivalent diameter $d = 2l/\pi^{\frac{1}{2}}$, where l is the length of the side of the plate (i.e. the diameter of a disk with the same area as the square plate). This observation is in line with the zero incidence results of Fail, Lawford & Eyre (1959) who concluded that the wakes behind bodies of small aspect ratio were essentially independent of the body shape. The Strouhal number for the square plate is also shown in figure 5.

$\omega°$	S	$-C_{pB}$	d'/d	S^*
0	0·136	0·364	1·70	0·198
10	0·150	0·408	1·65	0·208
15	0·158	0·428	1·60	0·212
20	0·170	0·452	1·49	0·210
25	0·186	0·491	1·35	0·206
30	0·206	0·512	1·20	0·201
35	0·238	0·545	1·10	0·210
40	0·275	0·650	0·95	0·203

TABLE 1. Variation of Strouhal number, base pressure and wake width with angle of incidence.

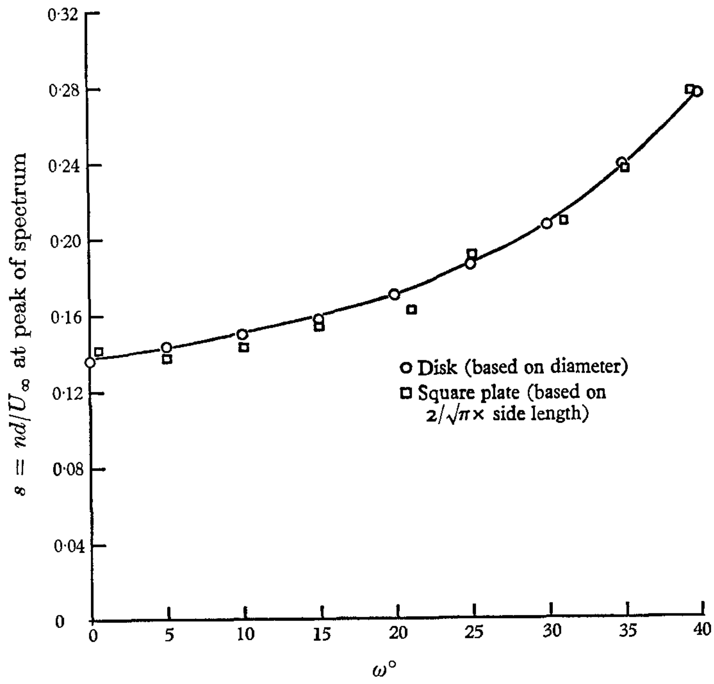

FIGURE 5. Variation of Strouhal number with incidence for square plate and disk.

Base pressure

The base pressure was measured by placing a Pitot tube at the centre of the rear surface as close to it as possible (the tube being normal to the surface). To be sure that the value obtained is representative of the average pressure over the rear surface it is necessary to know the pressure distribution over it.

With the disk at 30° incidence, measurements were made over a large part of the rear surface. The values of C_{pB} obtained ranged from $-0{\cdot}490$ to $-0{\cdot}545$, the average being about $-0{\cdot}517$. But these values were at isolated points. No systematic variation of pressure over the base could be detected with the technique used.

The accuracy of the base pressure measurements would be expected to improve as the distance between the probe and the surface is reduced, but because of the characteristics of the traverse gear, this distance could not be accurately measured or repeated within limits of about 0·005 to 0·010 in. A probe was therefore mounted on a micrometer head and the pressure variation close to the base at one point was measured. The pressure coefficient fell from $-0{\cdot}509$ at about 0·003 in. from the surface to about $-0{\cdot}541$ at 0·019 in. away. This variation is enough to account for most of the scatter of results mentioned above. It was

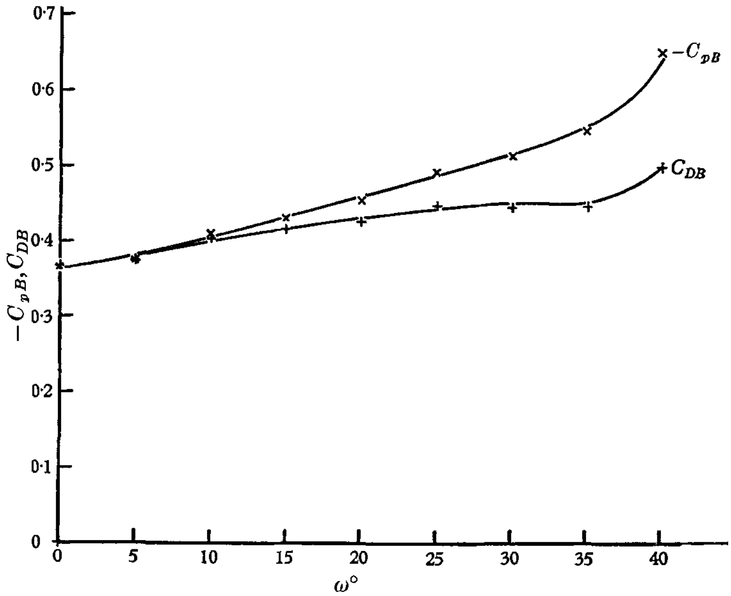

FIGURE 6. Variation of base pressure and base drag with incidence.

therefore decided that a representative value of base pressure would be taken as the pressure measured near the centre of the disk with the probe as close to the rear surface as possible. The value so obtained with the disk at 30° was $-0{\cdot}512$. In the only place where the base pressure has been used in calculation (the determination of the wake Strouhal number, see below) it appears as $(1-C_{pB})^{\frac{1}{2}}$, so that small errors in C_{pB} will not have too much effect.

The variation of base pressure with incidence is given in table 1 and figure 6. The base pressure falls with increasing incidence; this is consistent with two-dimensional flows where a strong vortex street is associated with low base pressure.

Also plotted on figure 6 is the base drag coefficient $C_{DB} = -C_{pB}\cos\omega$, which rises with incidence. According to Hoerner (1965) the total drag falls, so that the drag arising from the pressure distribution over the upstream face must fall as the incidence is increased.

Wake Strouhal number

It has been shown (Calvert 1967) that a Strouhal number may be defined in terms of quantities measured in the wake of a cone, and that such a Strouhal number has the value 0·19 irrespective of the angle of the cone. As a disk at zero incidence is a limiting case of both the family of cones and the family of inclined disks, it is reasonable to look for a corresponding wake Strouhal number for the latter set.

To non-dimensionalize a frequency, a velocity and a length are required. For the wakes of the cones the velocity chosen was that just outside the wake at the longitudinal position at which the static pressure on the centre line was a minimum. From the results of Fail *et al.* (1959) it was assumed that this velocity Um would be given by $Um/U_\infty = (1-C_{pB})^{\frac{1}{2}}$. The length chosen was the distance d' between the peaks of a traverse of $\theta = (\overline{u^2})^{\frac{1}{2}}/U_\infty$ in the y direction at this same position. Then the wake Strouhal number $S^* = S(d'/d)\,(1-C_{pB})^{-\frac{1}{2}}$.

For the inclined disk it was assumed that the same velocity $Um = U_\infty\,(1-C_{pB})^{\frac{1}{2}}$ would be relevant. The definition of d' is, however, more difficult. To define the position at which the y traverse should be made it is necessary to know the position of the static pressure minimum in the wake. It is not at all obvious that a traverse along the x axis will produce meaningful results, nor is there any clear alternative line along which to traverse. Accordingly, a more detailed investigation of the wake was carried out for the 30° incidence case.

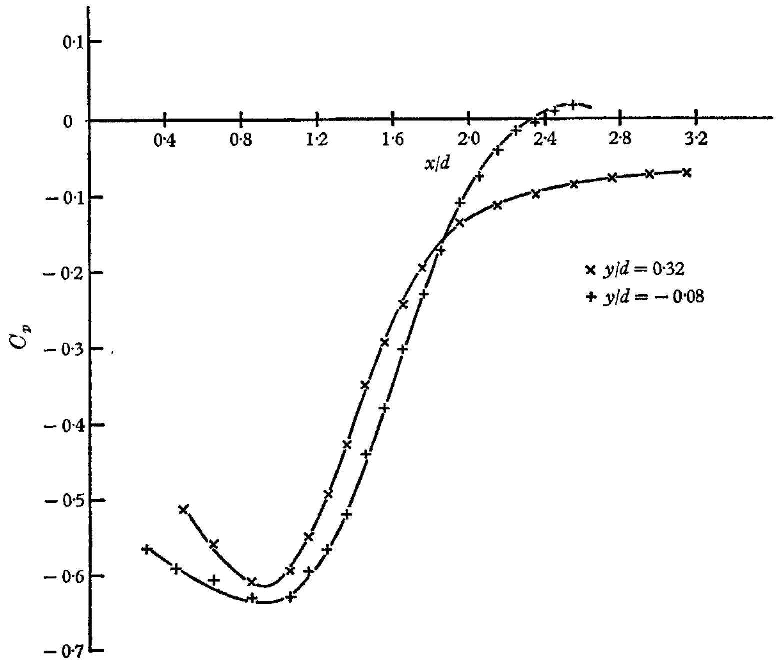

FIGURE 7. Static pressure variation behind disk at 30° incidence.

Velocity traverses in the (x, y)-plane showed that the mean velocity was minimum along the line $y/d = 0·32$ approximately, and a static pressure traverse along this line showed a clear minimum (figure 7). For comparison, a traverse was made along the line $y/d = -0{\cdot}08$ (above the support sting). This showed a minimum at about the same point, although the shape of the curve was somewhat different. These velocity and static pressure traverses are further discussed below.

To avoid the necessity of identifying the line of minimum velocity for all angles of incidence it was assumed that the traverse above the support sting would have a minimum at the same position as that along the line of minimum velocity, and that this is the relevant position to make a traverse in the y direction to identify d'. Additional justification is provided by the observation from the velocity traverses that the wake width is changing only slowly in this region, so that an error in the position of the pressure minimum will not lead to a large error in d'.

Traverses across the wakes at the position thus defined show two prominent peaks of turbulence, with two lesser ones between them. The distribution is unsymmetrical, the turbulence being much higher on the trailing side than the leading. At high angles of incidence the central minimum disappears and the two

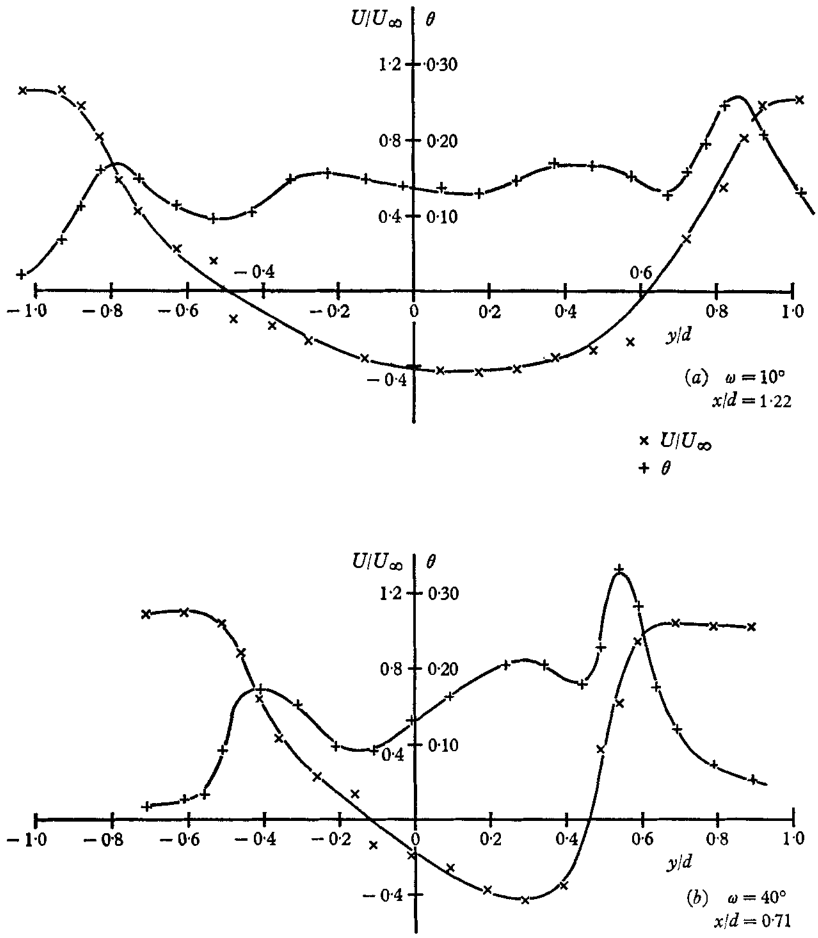

FIGURE 8. (*a*) Variation of mean velocity and turbulence across wake of disk at 10°, $x/d = 1{\cdot}22$. (*b*) Variation of mean velocity and turbulence across wake of disk at 40°, $x/d = 0{\cdot}71$.

inner maxima merge. At zero incidence the distribution is completely symmetrical. Two typical distributions of mean velocity and turbulence are shown in figure 8.

The wake width d' is defined as the distance between the two major peaks of the turbulence profile, and is given in table 1. (It was noticed that, as for the family of cones, these maxima were close to the points at which $U/U_\infty = 0{\cdot}6$.) For the limiting case of the disk at zero incidence this definition of d' leads to the same definition of S^* as that used for the wakes of cones.

The wake Strouhal number turns out to be almost the same for all angles of incidence; its values are given in table 1. The mean value is 0·206, with scatter +0·006, −0·008. This is higher than for the cones, where the corresponding figures were 0·191, +0·005, −0·004. The disk at zero incidence provides the lowest point (0·198) of the first set. This is just above the range of the second set. However, the value previously obtained for the disk was 0·187, which is definitely outside the range of the inclined disk values. (Scatter of this amount for one model was noticed previously when all the measurements for one cone were repeated.) It thus appears that the flow over the disk at zero incidence can be considered as the limiting case of the flow over the family of cones but not of the flow over inclined disks.

The reason for this is that the plane of vortex shedding is not defined in this case. When the vortices are being shed in the (x, y)-plane an unsymmetrical profile of θ will be obtained, somewhat similar to that for 10° incidence, figure 8*a*. When they are being shed in any other plane the profile will be different, although it will probably have the same basic shape with two high peaks at the sides and two lower ones in the centre. The orientation of the plane of shedding presumably varies randomly, so that the profile actually measured will be the mean of all these profiles. The distance between the peaks of this mean curve will not necessarily be the same as that between the peaks of the basic unsymmetrical curve which would be obtained with shedding consistently in the (x, y)-plane, and thus the value of S^* will be different.

It is not possible to check this explanation directly, as the vortex shedding from a disk at zero incidence cannot be persuaded to remain in one plane for long enough to take a traverse. Nor can the results from an inclined disk be used as the lift developed tends to displace the wake to one side. An approximate check can be obtained from the results at 10° incidence, where the lift is small. The line of minimum mean velocity for this traverse is at about $y/d = 0{\cdot}12$. If the θ profile is reflected about this line, and the mean of the original profile and the mirror image taken, the curve obtained is very similar to that at zero incidence. Its width d' is $1{\cdot}50d$ (previously $1{\cdot}65d$) which gives a value of S^* (using the 10° values of C_{pB} and S) of 0·189 (previously 0·208). This value is well within the scatter of values for the cones and the disk at zero incidence.

Static pressure traverses

As described above, it was desired to know at what longitudinal position the static pressure on the (as yet undefined) wake centre line was a minimum. Due to the design of the traverse gear and support system it was not practicable to traverse along the line $y = 0$, and in any case there is no reason to suppose that this is always the wake centre line.

A hot-wire investigation of the wake of the disk at 30° incidence (described below) showed that the mean velocity was minimum on the line $y/d = 0{\cdot}32$ approximately. A static pressure traverse was taken along this line; the results are shown in figure 7. There is a clear pressure minimum a little way downstream; the pressure then rises fairly rapidly and levels off to a value somewhat below the free-stream static pressure. The general shape of this curve is very much more like the pressure distribution along the centre line of the wake of a two-dimensional flat plate than that along the centre line of the wake of a disk normal to a stream (see, for example, Fail *et al.* 1959). The main differences between these two are that the pressure near the body is much lower in the two-dimensional than in the axisymmetric case (intermediate in this case) and that in the latter case it reaches a maximum, often above free-stream static pressure, and falls off again further downstream. In the former case there is no maximum, and the pressure levels off well below free-stream static pressure. This difference is thought to be related to the existence of a vortex street, so that the shape of the curve for the inclined disk is consistent with the strong vortex shedding in its wake.

Measurements at several angles of incidence showed that the minimum of the static pressure traverse was at about the same position irrespective of the y position of the traverse. To find the position of the minimum, therefore, traverses were made along the line $y/d = -0{\cdot}17 \sin\omega$ (the line above the support sting, which is at $z/d = -0{\cdot}1$). This line was chosen purely for convenience and has no physical significance. The main difference between these curves and those on the line of minimum velocity is that they rise to approximately the free stream pressure, sometimes having a slight maximum. The curve for 30° incidence is shown on figure 7; it will be seen that despite its different shape, its minimum is very close to the same position as that of the curve along $y/d = 0{\cdot}32$.

Velocity and turbulence measurements

Measurements were made of the longitudinal component of velocity and turbulence in the (x, y)-plane with the disk at various angles of incidence. For most angles the only traverse taken was in the y direction at the longitudinal position of the static pressure minimum. The results are described above and two typical traverses are shown in figure 8. For the 30° case, an exploration of the wake on a $0{\cdot}1d$ grid was made in the region of $0{\cdot}65 \leqslant x/d \leqslant 2{\cdot}35$ and $-0{\cdot}7 \leqslant y/d \leqslant 1{\cdot}3$. The results are shown as contours of mean velocity in figure 9. There is a region of reversed flow extending to about 1·7 diameters behind the disk. The wake is displaced towards the trailing edge side and the shear layer on this side is much thinner than on the other.

Figure 10 shows mean velocity and turbulence profiles across the wake at three different longitudinal positions. Figure 10*a* is near the position of the static pressure minimum. It will be seen that the central minimum of the θ

traverse visible on the 10° profile (figure 8*a*) but not on the 40° profile (figure 8*b*) is here just visible as an inflexion. The mean velocity curve is distorted towards the trailing edge side, with a very much thinner shear layer on this side than the

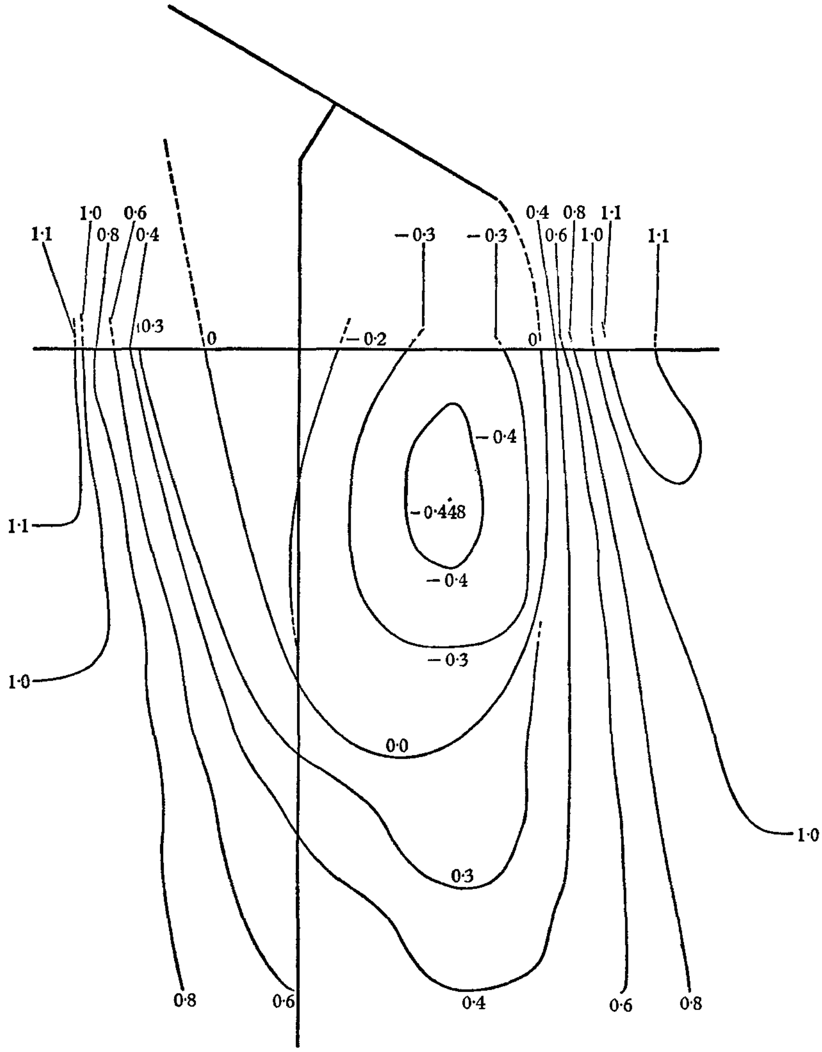

FIGURE 9. Contours of mean velocity behind disk at 30° incidence.

other. Further downstream (figures 10*b* and 10*c*) the general shape is much the same. The velocity deficit becomes less, but remains unsymmetrical. The turbulence distribution becomes more uniform, although the height of the peaks remains much the same.

A few traverses were made in the z direction. These were in all cases symmetrical about $z = 0$. The mean velocity and turbulence profiles had the same general shape as those in the y direction. For all the hot-wire measurements the region in which the mean velocity is negative must be decided by inspection, as the anemometer can give no information on this point.

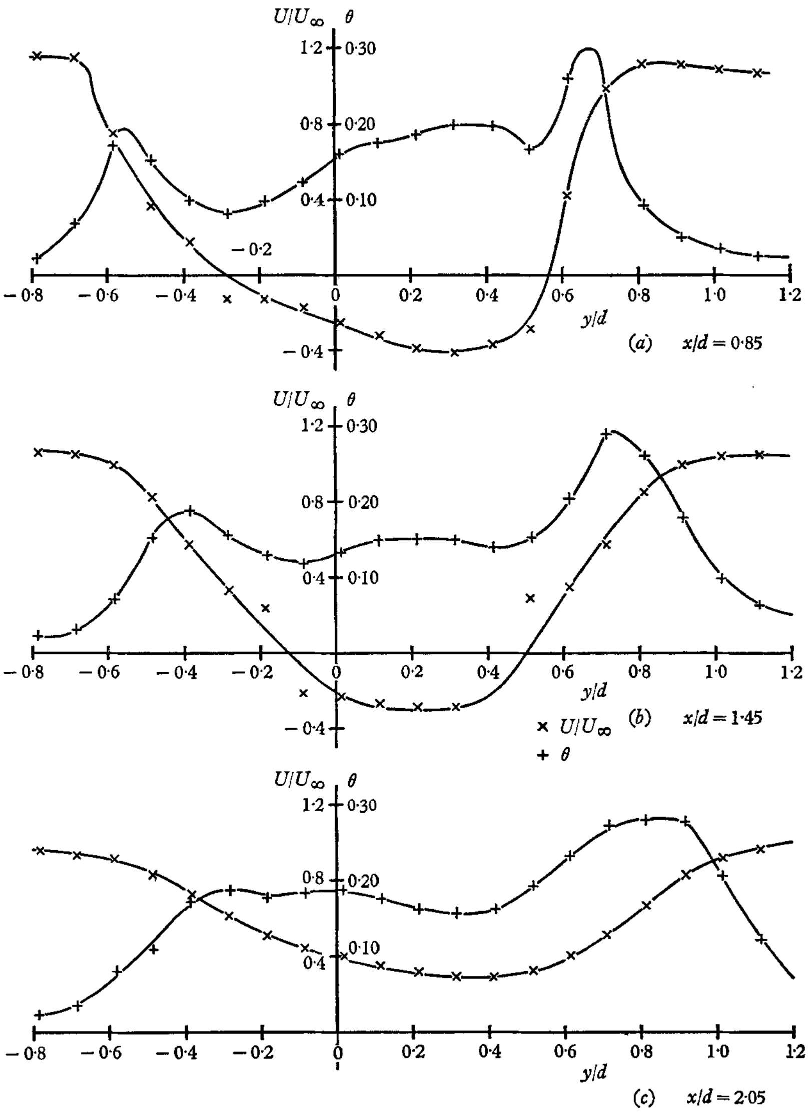

FIGURE 10. Profiles of mean velocity and turbulence across wake of disk at 30° incidence: (*a*) $x/d = 0{\cdot}85$, (*b*) $x/d = 1{\cdot}45$, (*c*) $x/d = 2{\cdot}05$.

REFERENCES

CALVERT, J. R. 1967 Experiments on the low speed flow past cones. *J. Fluid Mech.* **27**, 273.

EDEN, C. G. 1912 Investigation by visual and photographic methods of the flow past plates and models. *A.C.A. R & M* 58.

FAIL, R., LAWFORD, J. A. & EYRE, R. C. W. 1959 Low speed experiments on the wake characteristics of flat plates normal to an air stream. *A.R.C. R & M* 3120.

HOERNER, S. F. 1965 *Fluid Dynamic Drag*. Published by the author (p. 3. 16).

ROSENHEAD, L. 1932 Appendix to Stanton & Marshall (1932). *A.R.C. R & M* 1358.

STANTON, T. E. & MARSHALL, D. 1932 On the eddy system in the wake of flat circular plates in three dimensional flow. *A.R.C. R & M* 1358. Also 1931 *Proc. Roy. Soc.* A **130**, 295.

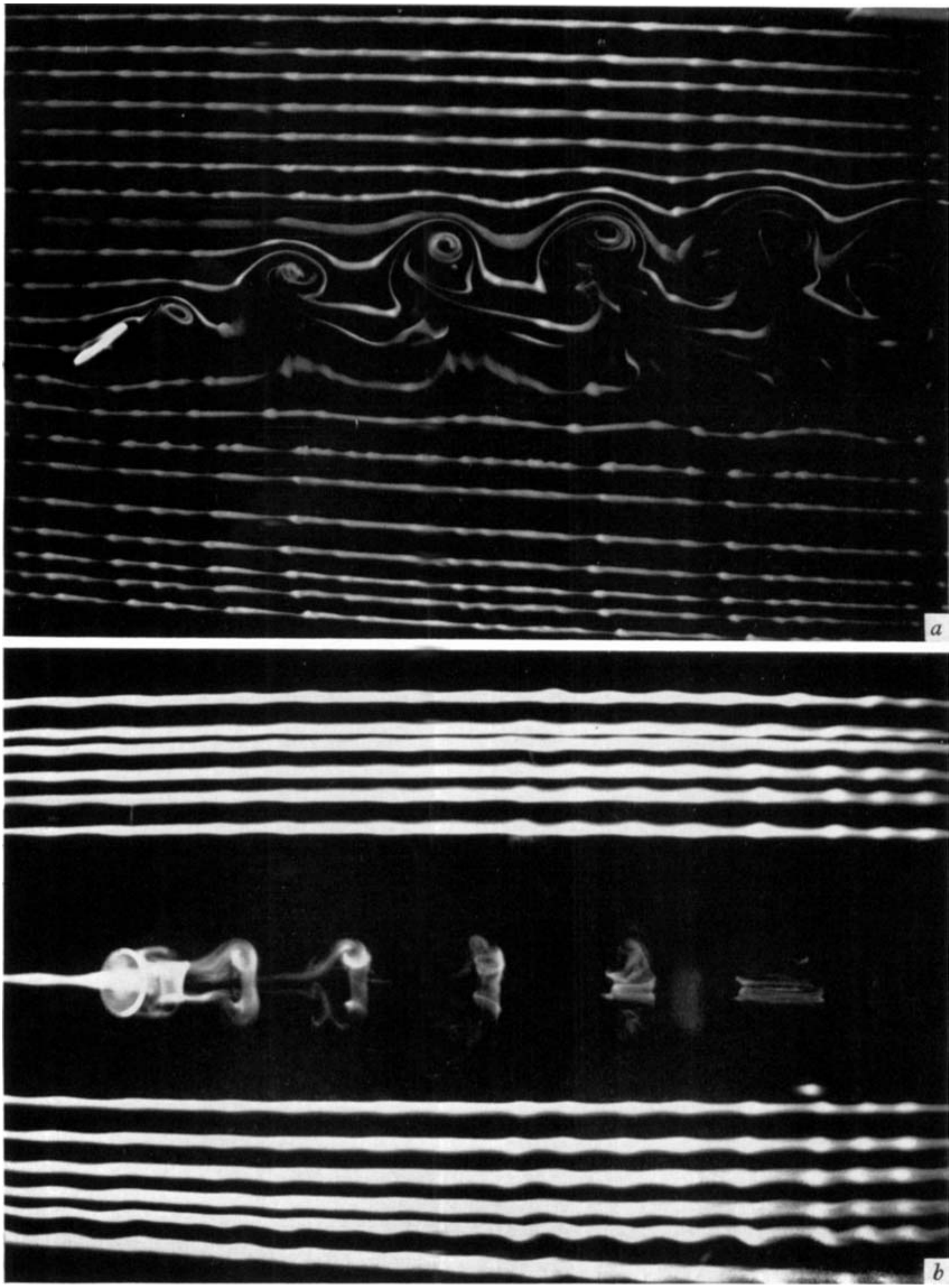

FIGURE 2. (Plate 1) Vortex chain behind disk at about 50° incidence. (*a*) (x, y)-plane, (*b*) (x, z)-plane.